On the Figure of Merit Model for SEU Rate Calculations

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Abstract

Petersen has introduced a one parameter characterization of a device by the Figure Of Merit (FOM). It was claimed that this parameter was sufficient to estimate the SEU rate in almost all orbits. The present paper presents an analytic study of the FOM concept and compares the FOM model with other empirical models. It is found that indeed the *FOM* parameter gives, in most cases, a good agreement with the rates found using the full SEU cross section plots of the devices. The agreement is poorer in cases where a high portion of the proton flux comes from low energy protons and for very SEU-hard devices. This is demonstrated for certain devices (FPGAs) where the *FOM* predicted by proton may be smaller by an order of magnitude than the *FOM* from heavy ions.

I. Introduction

The prediction of single event upset (SEU) rates R of a given device in space is accomplished by measuring its cross sections for heavy ions and protons and folding them with ion fluxes in the given orbit. Many methods, like calculating the proton induced SEU (p-SEU) cross sections from heavy ion data (see examples in Section IV), have been presented to deal with partial information on those cross sections. The method introduced by Petersen [1] and discussed here is particularly interesting due to its simplicity and the large number of cases which validated it.

Petersen has demonstrated in [1] how his earlier defined parameter (of each device), the figure of merit (FOM), can serve as a single number to estimate the device SEU rates in almost all orbits. This is an unexpected conclusion in view of the variety of possible cases: (i) devices have different critical charge, geometry of sensitive volumes, etc., (ii) orbits have different spectra of linear energy transfer (LET or L) and proton energies and (iii) different shielding changes the spectra. The observation that all these variations are taken care of by the *FOM* parameter and the orbit constant C calls for studying the basic mechanisms.

The FOM model and its limitations were studied by Petersen in [2]. He used generic device characteristics and compared the FOM predictions with integral rectangular parallelepiped (IRPP) results. In [1] Petersen compared the results of his model for actual devices with those calculated using IRPP codes (Space Radiation and CREME96 with Weibull parameters).

In the analysis here we (like Petersen) use CREME96 [3] (solar minimum and solar quiet) for calculating the flux and (like in [4]) fitting it to power law curves. We find analytic forms for the upset rates for easier evaluation of the model.

Petersen also showed [1] how to derive *FOM* from experimental proton induced SEU cross section: the *FOM* is just proportional to the limiting p-SEU cross section, σ_{pL}. We find that indeed σ_{pL} can be used for calculating p-SEU rates and heavy ion SEU rates in ordinary devices. However, using $FOM \propto \sigma_{pL}$ for devices with high critical LET (L_c) would lead to an over-estimation of the p-SEU rates and an underestimation of heavy ion SEU rates. We also give an experimental example for this case by examining the *FOM* computed from proton data and heavy ion data independently.

For usual devices, the proportionality of σ_{pL} to the *FOM* forms, in fact, a new empirical model for predicting σ_{pL} from heavy ions cross sections. This issue will be discussed in detail.

II. Heavy Ion SEU Rates Using FOM

A. General Definition of the FOM for Heavy Ions

Introduced by Petersen [1,2,5,6] the *FOM* was defined as

$$FOM \equiv \sigma_{HL}/(L_{0.25})^2 \quad . \tag{1}$$

σ_{HL} is the limiting (saturation) value of the heavy ion cross section per bit, $\sigma(L)$, given in units of cm^2 (per bit). $L_{0.25}$ (in MeVcm^2/mg) is the value for which $\sigma(L_{0.25})=0.25\sigma_{HL}$.

The use of *FOM* for SEU rate calculations was based on a simplified LET spectrum and a simplified RPP geometry [2]. Petersen has shown in [1] that for most orbits

$$R = C \times FOM \quad . \tag{2}$$

C (the rate coefficient), given in upsets/bit-day, is an orbit parameter. It depends only on (i) the orbit, (ii) the shielding, (iii) the kind of ions (protons, heavy ions) and (iv) the device hardness (ordinary or SEU-hard).

The heavy ions SEU rate, R_H, is usually calculated by

$$R_H = \int_0^\infty f(L)\ \sigma_e(L)\ dL \quad . \tag{3}$$

$\sigma_e(L)$ is the 'effective' cross section, i.e. the cross section averaged on all solid angles (see Appendix). $f(L)$ is the heavy ions differential ion-flux spectrum in a given orbit and for a given shielding.

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B. Analytic Evaluation of the FOM Computed from Heavy Ion Data

R_H is expected to decrease with increasing $L_{0.25}$ but the simple proportionality to $L_{0.25}^{-2}$, with no adjusting parameters, is not simple to accept. As shown in [4,5,7] and below, it is possible to approximately arrive at this dependence for heavy ions whose $f(L)$ goes like L^{-3} and for a simple function for the cross section (so that the integration on L brings R_H to a L^{-2} dependence, see eqs. 4, A1).

For calculating R_H we find $f(L)$ using standard codes. $f(L)$ units used here are ions/(cm^2 day MeVcm^2/mg). The output of CREME96 is given in ions/(m^2 sr sec MeVcm^2/g) thus its differential flux output must be first converted by multiplying it by 108573. Many spectra were presented in [8]. Here we use CREME96 with all ions (z=1 to 92).

It was observed earlier [4] that the LET range for ions can be divided into sections in which $f(L)$ is presented by power laws. Fig. 1 gives $f(L)$ for the galactic cosmic rays (GCR). With a shielding as small as 40 mils (~1mm), which already prevents heavy ions with low energy from reaching the devices, $f(L)$ may be fitted amazingly well by three functions, $f(L)=pL^{-n}$, for three domains (p and n are given in Fig. 1). Many devices have their critical LET between 1 and 28 MeVcm^2/mg (we defined the latter as L_{max}), making this the most interesting LET regime. For lower LET values (relevant for devices with very low L_c) and higher values (for very high L_c) the flux is much weaker and the SEU rates are expected to fall well below those calculated by eq. (2) using the C value for usual devices.

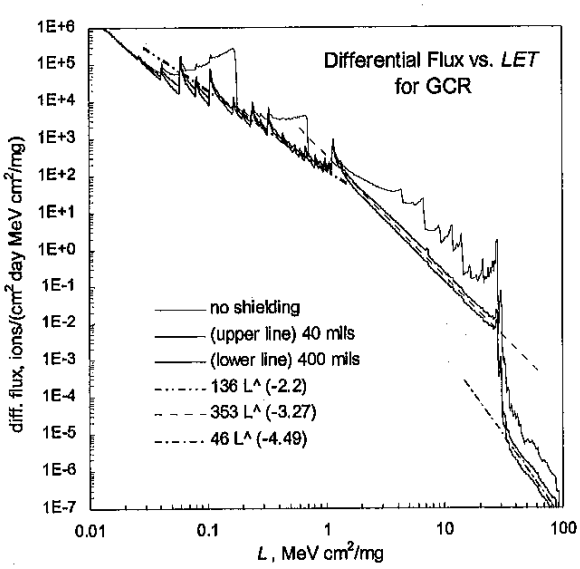

Fig. 1. Differential flux *vs* LET spectra of GCR, for no shielding, behind 40 mils and behind 400 mils of aluminum. The three power law fits in the three domains, $f(L)=pL^{-n}$, are for a shielding of about 200 mils.

For our calculations we take a disc with diameter a, a thickness c and a high aspect ratio, $c \ll a$. The device has a critical energy ε_c. The exact chord length distribution for this case was given in [9]. Here we use an approximated evaluation. For a beam at an angle θ (taken with respect to the normal to the disc plane), the ion track is $l=c/\cos\theta$ and the cross section is $\sigma_\theta=S\cos\theta$, where $S=\pi a^2/4$. The energy deposited by an ion is $\varepsilon=\rho cL/\cos\theta$. ρ is the density (2.32 g/cm^3 for silicon). The normal incidence $\sigma(L)$ defines a critical LET, L_c, below which there will be fewer events (25% for $L_{0.25}$). $\varepsilon_c=L_c\rho c$. For simplicity we use a step function response, i.e. there will be an event if $L\rho l>\varepsilon_c$.

We use eq. (3) with the power law for $f(L)$ and with a cuttoff at L_{max}=28 MeVcm^2/mg. We first integrate on L with $f(L)\sigma_\theta=f(L)S\cos\theta$ for $L\geq L_c\cos\theta$ and 0 otherwise. The integration on θ starts from θ=0 for $L_c<L_{max}$ and from $\cos\theta=L_{max}/L_c$ for $L_c\geq L_{max}$:

$$R_H = \int_{\cos\theta=L_{max}/L_c}^{\theta=\theta_{max}} S\cos\theta \left[\int_{L_c\cos\theta}^{L_{max}} pL^{-n}dL \right] d(-\cos\theta) \quad . \quad (4)$$

θ_{max} was introduced since the maximum chord length is ~a thus $\cos(\theta_{max})\approx c/a$.

For $L_c<L_{max}$ and $n>3$ we have

$$R_H = \frac{p[(a/c)^{n-3}-1]}{(n-1)(n-3)} \frac{S}{L_c^{n-1}} - \frac{p[(1-c/a)^2]}{2(n-1)} \frac{S}{L_{max}^{n-1}} \quad . \quad (5)$$

For $L_c \ll L_{max}$ and the actual $f(L)=353L^{-3.27}$ (p=353, n=3.27, Fig. 1) we find $R_H=C^*S/L_c^{2.27}$ with $C^*=576[(a/c)^{0.27}-1]$. Replacing L_c by $L_{0.25}$ it is clear that R_H behaves close to Petersen's formula (eq. 2). For $a/c\approx10$, it is close to the values C=485 or 360 in Fig. 1 of [1]. C^* slowly decreases with a/c.

Modern devices might have $a\approx c$ and the above approximation does not hold. We recalculate R_H for this case by considering $\sigma=S$ in all directions. This yields

$$R_H = \frac{pS}{n-1}\left(\frac{1}{L_c^{n-1}} - \frac{1}{L_{max}^{n-1}}\right) \quad . \quad (6)$$

Here C^*=156 which is smaller than in [1]. For such devices the FOM model overestimates R, as is evident in the curves for CS=1E-7 in Fig. 6 of [2].

C. Discussion of Results

Eqs. (5,6) show that, for the central domain in Fig. 1, increasing L_c towards L_{max} or even passing it ($L_c>L_{max}$ in eq. 4) will decrease the SEU rate much faster than expected from the FOM formula. Petersen [1] showed (using Space Radiation and CREME96) that this could be compensated for by reducing C by a factor of 4. On the other hand the flux falls at 28 MeVcm^2/mg by more than two orders of magnitude (to the third domain in Fig. 1). To explain this apparent discrepancy we might see (Table 1 in [1]) that the devices with $L_c\geq28$ MeVcm^2/mg had a»c. The higher 'effective LET' of the heavy ions, when hitting at grazing angles, enables them to induce

SEU. This is formally shown in eq. (A3). See also the fit of the FOM method for generic devices (Fig. 6 in [2]).

The other extreme is that of devices with $L_c \leq 1$ MeVcm^2/mg. Fig. 1 shows that below this LET, $f(L)$ is much below the extrapolated line of the middle domain. For such devices, the FOM model would overestimate R_H. As an example for this case we might take single event transients in photodetectors [10].

The above analysis was for GCR. It is good for all orbits which have a similar shape of $f(L)$, i.e. $f(L)$ behaves as a power law. Ref. [8] examined many orbits and shieldings. It showed (Figs. 22, 23 of [8]) that (due to the magnetosphere shielding) the above main power law region shrinks for orbits with both inclination less than 50° and altitude below 10000 km. Ref. [1] gives the Rate Coefficient (C) values for such orbits but the standard deviation of the fit is worse here. It should however be noted that, for these orbits, most SEU events, for devices with $L_c \leq 10$ MeVcm^2/mg, are expected to come from protons and the exact calculation of heavy ion induced SEU rates is less important.

III. Calculating R for Protons

A. General Definition of the FOM for Protons

For protons eq. (3) is replaced by

$$R_p = \int_0^\infty f(E_p)\ \sigma(E_p)\ dE_p \tag{7}$$

where E_p are the energies of the protons.

$\sigma_p(E_p)$ is best determined by measuring its values in some proton energies and fitting the results to an analytic function like Bendel, Weibull, or integral log-normal. The curves are characterized by the limiting value, σ_{pL}, and the 'critical' value E_{pc}, at which $\sigma_p(E_p)$ reaches (say 50% of) the saturation value (σ_{pL}). No angular corrections are needed since the protons impinge the device omnidirectionally and since most devices have only weak anisotropy.

Petersen found that the FOM is proportional to σ_{pL} [1]

$$FOM = 4.5\times10^4 \sigma_{pL} \tag{8}$$

With eq. (2), $R=4.5\times10^4 C\sigma_{pL}$, which means that, in theory, both proton- and heavy ion SEU rates can be calculated using the p-SEU limiting cross section alone. We only need to know, for the given orbit, its C value for protons and its C for heavy ions. As is shown below, for p-SEU rates this is in general true since most protons reaching a device in orbit have $E_p > E_{pc}$. Ref. [1] shows that this type of analysis is valid for conservative estimates. For certain cases, however, it will be shown, later in this paper, that these results are not general.

B. Analytic Evaluation of the FOM Computed from Proton Data

The spectrum of the protons trapped in the belt includes a high flux of low energy protons but a shielding as thin as 40 mils of aluminum stops most of them. This is demonstrated in Fig. 2 by the integral flux plots for a 600×1000 km 60° orbit. For 40 mils shielding the median energy of $f(E_p)$ is 70 MeV and for 200 mils it is 97 MeV. For devices with E_{pc} smaller than these numbers, like those presented in [1], there will be at most an error of ×2 if we take $\sigma(E_p)=\sigma_{pL}$. Doing so in eq. (7) gives $R_p = A\sigma_{pL}$ where A is simply the total flux of protons in a given orbit and behind a given shielding. A is just the value at $E_p=0$ of plots like those in Fig. 2.

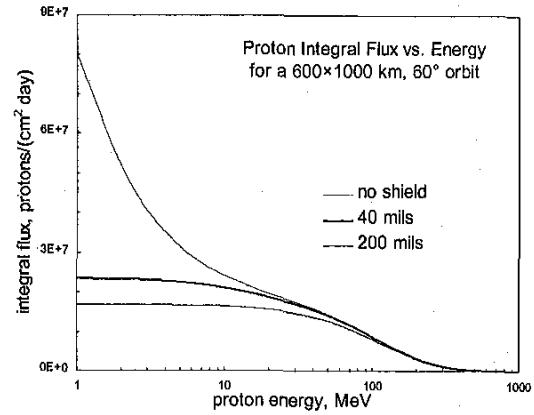

Fig. 2. Proton integral flux (solar minimum) for different thicknesses of shielding.

For a better estimation only protons with $E_p > E_{pc}$ are considered. This means that we approximate $\sigma_p(E_p)$ in eq. (7) by a step function with the full value, σ_{pL}, above E_{pc}. This adds a factor of efficiency F to the flux A:

$$R_p = FA\sigma_{pL} \tag{9}$$

For estimating F we plot in Fig. 3 the normalized integral flux *vs* E_p for several orbits which were treated in [1]. For the actual flux, each plot should be multiplied by its A value (before the flux was normalized, see Table 1). It is found that the normalized functions depend essentially only on the altitude while A is determined also by the inclination. For altitudes above 3000 km the averaged protons energy decreases fast and their 'efficiency' to trigger SEU is high only for soft devices (low L_c gives low E_{pc}, see section V). This is the reason for the worse applicability of the FOM method for protons at high altitude [1].

Fig. 3 shows that in all orbits there only a small portion of protons have E_p>200 MeV. Thus a measured σ_p at this energy might be used for σ_{pL}.

Fig. 3 also includes the plot for the MPTB orbit. This is a 1220×39200 km 63.6° orbit and the shielding is a 60 mil aluminum plate on one side (with very high shielding on the other side). The integral flux is $A=4.1\times10^7$ p/(cm^2 day). Further details on the calculations for this orbit will be given elsewhere.

Next, we find out what value of E_{pc} is the one which best reproduces the C values stated in [1] for the FOM model. We

use (see Fig. 3) E_{pc}=70 MeV and E_{pc}=35 MeV. The latter is more appropriate for the devices studied by Petersen [1]. From eqs. (2,8,9) C=2.22×10^{-5}FA. This gives the calculated C values in Table 1. Petersen's C values seem to be a bit smaller (by a factor less than two) for most cases.

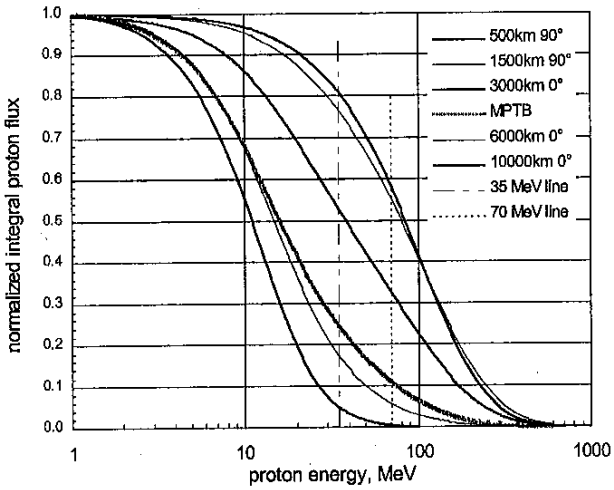

Fig. 3. The normalized (to 1) integral proton flux (solar minimum) behind 100 mills of Al. It gives the portion of the protons having energy higher than a given energy. For two E_{pc} values (35 and 70 MeV) the plot gives the F values in Table I. This is the probability of the nuclear reactions p+Si to deposit enough energy in the sensitive volume to trigger an SEU. The order of the orbits from above is as given in the legend (altitude and inclination),

Table 1.

A comparison of the C value of eq. (2), using C=2.22×10^{-5}FA, with that in [1] for two E_{pc} values. A is the total proton flux behind a shielding of 100 mil and F is the efficiency factor (see Fig. 3). C is in units of (bit day)$^{-1}$.

Altitude (km)	500	750	1500	3000	3000	6000	10000
Inclination	90°	90°	90°	90°	0°	0°	0°
Total Flux (A) (cm^2 day)$^{-1}$	1.86 ×10^6	9.65 ×10^6	1.46 ×10^8	7.67 ×10^8	3.70 ×10^9	1.52 ×10^9	9.04 ×10^7
(F) for E_{pc}^a=35MeV	0.810	0.806	0.766	0.529	0.545	0.174	0.049
Calculated C E_{pc}^a=35MeV	33	173	2483	9007	44766	5871	98
(F) for E_{pc}^b=70MeV	0.580	0.585	0.550	0.325	0.346	0.056	0.004
Calculated C E_{pc}^b=70MeV	24	125	1783	5534	28420	1890	8.0
Petersen's C	18	93	1324	4370	22300	2264	35

The above expression, R=4.5×10$^4C\sigma_{pL}$, suggests a way to estimate heavy ion SEU rates from proton induced cross sections. In the cases where the expression holds it would save heavy ion SEU tests. The analysis here indicates that the method would work well for usual devices ($L_{0.25}$≤10 MeVcm^2/mg, E_{pc}≤80 MeV, see section V).

Very few studies (e.g. [11,12]) dealt with estimating heavy ion cross sections from p-SEU cross sections, none claiming to have a good solution. The above expression for R means, in fact, that to estimate R_H of heavy ions from p-SEU data we do not need to first calculate $\sigma(L)$.

In very hard devices eq. (8) may not hold. This is demonstrated for three FPGAs studied in [13]. The Weibull parameters of these devices are given in Table 2 as well as the corresponding FOM and σ_{pL}. It is shown that the measured σ_{pL} might be more than an order of magnitude smaller than calculated using eq. (8). Alternatively, FOM from the measured σ_{pL} is much smaller than from measured $\sigma(L)$. This shows that to predict a conservative estimate of SEU rates for these devices both proton and heavy ion testing is required.

Table 2.

The table displays the following parameters for three SEU-hard FPGAs [13]: the Weibull parameters L_{th}, w, s and σ_{HL}, $L_{0.25}=L_0+w\times0.288^{(1/s)}$, the FOM from $\sigma(L)$ (calculated using eq. 1), the corresponding σ_{pL}, calculated using eq. (1), the measured σ_{pL} are (taken from Table 1 of [13]), and the corresponding FOM calculated using eq. (8). There is a disagreement between the FOM values calculated using heavy ion results and those using proton results. The disagreement also shows when comparing the calculated and measured σ_{pL} values.

device	RT54SX16	A1280A	A54SX16
L_{th}, MeVcm^2/mg	9	2	16
w, MeVcm^2/mg	30	30	37
s	2	2	1.5
σ_{HL}, cm^2/f-f	2.5×10^{-6}	6×10^{-6}	8×10^{-7}
$L_{0.25}$, MeVcm^2/mg	25.1	18.1	32.1
FOM from $\sigma(L)$	4.0×10^{-9}	1.83×10^{-8}	7.75×10^{-10}
calc. σ_{pL}, cm^2/f-f	8.8×10^{-14}	4.1×10^{-13}	1.72×10^{-14}
meas. σ_{pL}, cm^2/f-f	6×10^{-15}	1.4×10^{-13}	3×10^{-15}
FOM of meas. σ_{pL}	2.7×10^{-10}	6.2×10^{-9}	1.35×10^{-10}

IV. Estimating p-SEU Cross Sections Using Heavy Ion SEU Data

In the previous section we discussed the applicability of p-SEU cross sections to heavy ion rate estimations. Here we examine the opposite case. Since $R\propto\sigma_{pL}$ we only need to find σ_{pL} from heavy ion cross sections as done in Table 2. Using eqs. (1,8), it is given by

$$\sigma_{pL} = 2.22\times10^{-5}\,\sigma_{HL}/(L_{0.25})^2 \qquad (10)$$

This expression presents a simple method to derive σ_{pL} from $\sigma(L)$. It forms a new empirical model. We will analyze why and when it works and compare it with other empirical models.

There are several physical models, based on the physics of proton induced energy deposition in silicon, which calculate

$\sigma_p(E_p)$ from $\sigma(L)$. The most recent ones were published in [14-20]. The calculations, performed using Monte-Carlo simulations, give good agreement with experimental results. They might use the full $\sigma(L)$ curve (described, for instance, by a Weibull function) but with only the parameters σ_{HL} and L_c (and the sensitive volume thickness, c) they already yield good estimation of $\sigma(E_p)$. The drawback is first, that most of them (except for module 225 of the Space Radiation code and the CUPID/COSMIC codes [17]) are not available commercially and second, that the calculations are long and sometimes tedious. This motivated studies aimed at developing empirical models like eq. (10). We will compare this expression with other models.

First, we note that the BGR model was shown in [14] to yield good agreement with the data in [21]. These experimental data of both $\sigma(L)$ and $\sigma_p(E_p)$ for the same devices served Petersen (along with other data, see Fig. 2 in [1]) to establish the above eq. (8) (eq. 4 in [1]).

Next we use a physical semi-empirical model which utilizes experimental proton induced SBD spectra to calculate the energy deposition in the sensitive volume [22]. Protons deposit energy in the sensitive volume through nuclear-reactions with the material (silicon) nuclei. The recoils of the reactions ionize the material along their tracks. The probability to deposit an energy of at least $\varepsilon_c=\rho c L_c$, needed to trigger an SEU, is approximately an exponential function of ε_c. Following Ref. [22], for a step function for $\sigma(L)$

$$\sigma_p(E_p) = \alpha c \sigma_{HL} \; e^{-\beta(E_p)\rho c L_c} \quad . \qquad (11)$$

We take (like in [21]) the σ_p value at 200 MeV as the limiting value, $\sigma_{pL}=\sigma_p$(200 MeV), and c=2μm. Also β(200 MeV) ≈0.382 MeV^{-1} and α=0.02 cm^{-1} [22]. Using a 'correction factor' of 0.6 (which has led to better predictions of this model) we get

$$\sigma_{pL} = 2.4\times10^{-6}\sigma_{HL} \; e^{-0.177L_c} \quad . \qquad (12)$$

Though eqs. (10) and (12) look entirely different, equating L_c with $L_{0.25}$ we found that they agree to a factor of 2 for $3\le L_c\le 28$ MeVcm^2/mg (which covers most of the central domain in Fig. 1). For devices with $L_c\le 3$ MeVcm^2/mg or $L_c\ge 28$ MeVcm^2/mg, eq. (10) would overestimate σ_{pL} as shown for the hard devices in Table2. This means that using p-SEU results will underestimate the *FOM* and the heavy ion rates calculated with this *FOM*. For devices with high L_c, there will be a certain compensation since the FOM model uses for the high LET the power law of the center domain (Fig. 1), which causes overestimation (see above) even after the with a ×¼ factor.

Next, we compare eq. (10) with other empirical models. Such a model, which might be brought to a form similar to eq. (10), was presented in [23]. Eq. (2) there said that

$$\sigma_p(E_p) = 10^A W^B \quad . \qquad (13)$$

W was defined as $W=E_{th}/V$ where V is the volume of the sensitive volume (in μm^3) and E_{th} is the threshold energy for SEU (in MeV). W is in mJ/cm^3. A and B are parameters that depend on E_p. If we take for σ_{pL} the values $\sigma_p(E_p)$ at E_p=200 MeV then (their Table I) $A\approx$-11.9 and $B\approx$-1. Also $V=\sigma_{HL}c$. With $L_{th}=E_{th}/\rho c$ and after proper unit conversions

$$\sigma_{pL} = 3.39\times10^{-6}\,\sigma_{HL}/L_{th} \quad . \qquad (14)$$

For eq. (14) to agree with eq. (10) it requires that $L_{0.25}=2.55\,(L_{th})^{1/4}$. If σ(L) is given by a Weibull function then $L_{0.25}=L_0+w\times0.288^{(1/s)}$, where L_0 (=L_{th}), w and s are the Weibull parameters [1]. For certain sets of parameters one may find agreement. However, another option, discussed in [1], is to take $L_{0.25}=2.77L_0^{0.88}$. This should yield similar results to [23] as long as L_0 is close to 1 MeVcm^2/mg.

Another empirical model was presented in [21]. They use $\sigma(L)$ at L=15 MeVcm^2/mg which is the maximum LET value of the heaviest (silicon) recoil in the p+Si reactions. They also assume that c=2μm. Their eq. (15) is

$$\sigma_{pL} = 6\times10^{-7}\times\sigma(L=15) \quad . \qquad (15)$$

Using the Weibull equation for $\sigma(L)$, eq. (15) becomes $\sigma_{pL}=6\times10^{-7}\sigma_{HL}\{1-\exp(-[(15-L_0)/w]^s)\}$. For <u>$L_0\ll15$</u> MeVcm^2/mg and <u>large w</u>: $\sigma_{pL}\approx6\times10^{-7}\sigma_{HL}\times(15/w)^s$. This expression will be consistent with the above eq. (10) if <u>$s\approx2$</u> (a common case) and if we replace $L_{0.25}$ by 0.406w. On the other hand for these <u>underlined</u> conditions the Weibull fit yields $L_{0.25}$=0.537w and expressions (10) and (15) agree within a factor of ≤2.

We have compared Petersen's expression, eq. (10), with other models and have found mathematically when they agree. This is important since each model was shown, by its developer, to be in good agreement with experimental results.

V. DISCUSSION

An important issue in analyzing the FOM model is what we mean by a hard and soft (for SEU) device. The 'hardness' should reflect predicted SEU rates in space. The *FOM* is a good measure of the rates per bit. Low *FOM* can be accomplished either by reducing the bit cell size (or rather its sensitive volume) or by increasing its L_c. We think that the SEU rate per device is more important for system designing. It is given by *FOM*×(number of bits) i.e. by the limiting cross section per device divided by L_c^2.

If we increase L_c and the sensitive volume lateral dimensions by the same ratio, the *FOM* stays the same. As found above, having high lateral dimensions means better applicability of the FOM model for high L_c values. This explains why devices with low and high L_c values fit the same upset-rates *vs FOM* plots in [1].

There is a symmetry between the expressions (eqs. 3 and 7) for calculating R for heavy ions and protons (except for the angular dependence of the heavy ion cross section). $\sigma(L)$ is characterized by σ_{HL} and $L_{0.25}$ while $\sigma_p(E_p)$ by σ_{pL} and E_{pc}. Thus, it is interesting that the *FOM* value calculated from $\sigma(L)$

(eq. 1) takes both σ_{HL} and $L_{0.25}$ into account whereas the *FOM* calculated from $\sigma_p(E_p)$ (eq. 8) uses only σ_{pL}.

As was discussed above, σ_{pL} is indeed sufficient for devices with a small value of L_c. The reason for it is the much weaker dependence of $f(E_p)$ on E_p (Figs. 2,3) as compared to that of $f(L)$ on L (Fig. 1). Yet, for large-L_c devices we should add a reduction factor (which would take E_{pc} into account) to eq. (8), as was shown in the above examples of Table 2.

The relation between E_{pc} and $L_{0.25}$ can be estimated using the model of [22]. For c=2μm (see eq. 11 above) we have

$$\sigma_p(E_p) = 2.4\times10^{-6}\sigma_{HL}\ e^{-\delta(E_p)L_c} \qquad (16)$$

with

$$\delta(E_p) = 0.134 + 9MeV/E_p \qquad . \qquad (17)$$

$\delta(E_p)$ is in units of (MeVcm^2/mg)$^{-1}$. For E_p=200 MeV, eq. (16) yields eq. (12). Taking the value of E_{pc} as the energy for which $\sigma_p(E_{pc})$=½σ_p(200 MeV) we find

$$E_{pc} = 9L_c / (0.043L_c + \ln 2) \qquad . \qquad (18)$$

For L_c>10 MeVcm^2/mg we have E_{pc}>80 MeV and eq. (8) would overestimate R unless a reduction factor, which depends on E_{pc}, is added.

In the present analysis we used a step function approximation for $\sigma(L)$. One parameter L_c (=$L_{0.25}$) stands for three Weibull parameters, L_{th}, w and s. This works well for most cases. It seems, however, that when the width of the $\sigma(L)$ (represented by w) is large, the approximation might be less good in particular when applying the *FOM* derived from heavy ions to p-SEU rates and vise versa. This point is demonstrated in Table 2.

As found in eq. (5) the power law in Fig. 1 predicts a $L_c^{-2.27}$ dependence of the heavy ion SEU rate. The FOM model uses a L_c^{-2} dependence. The deviation from the average value, in the limits of the central domain (1-28 MeVcm^2/mg), might be as much as the square root of $28^{0.27}$ which is a factor of 1.5. This factor adds to the overestimation of the SEU rates for large-L_c devices.

VI. Conclusions

The calculations presented here show that the FOM model is well established theoretically for the following cases:

(1) *For heavy ion induced SEU* in orbits out of the magnetosphere where the differential LET spectrum behaves close to a reciprocal cube power law. It is verified for devices with $(c/a)L_c$«28 MeV cm^2/mg. At low altitudes and inclinations the power law behavior is altered and we expect variations with the value of L_c.

(2) *For p-SEU* for L_c≤10 MeV cm^2/mg (E_{pc}≤80 MeV) and orbits not too high so that the number of low energy protons is small. For SEU-hard devices, deriving *FOM* from proton cross sections might lead to underestimating R for heavy ions.

We have found that for protons the FOM method presents an empirical model to calculate the limiting p-SEU cross sections from heavy ions results. This model was compared both in its form and results to other models. It gives good agreement for the data in [1].

The FOM method [1] was developed in order to enable estimation of the SEU rate with partial knowledge of the SEU cross sections. If there is enough (experimental or calculated) data on $\sigma(L)$ and $\sigma_p(E_p)$ of a certain device the rates might be directly estimated using CREME96 or Space Radiation codes. In this work we analyzed the FOM method and compared it with alternative approaches which use theoretical or empirical programs to complete the data set or the needed parameters before using the above codes for rate calculations.

Acknowledgments

We are indebted to J.L. Barth and J.W. Howard for fruitful discussions and technical help. We would like to thank E.L. Petersen for his critical reading the manuscript and adding his insight of the subject through his comments and discussions.

One of the authors (J. Barak) is very grateful for the hospitality and support of NASA/Goddard Space Flight Center and of the National Research Council. This work was performed while he held a NRC-NASA/GSFC Research Associateship.

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APPENDIX

In Sec. II we calculated R of eq. (3) by first integrating on L and then on θ (eq. 4). Since we use an approximated function for $\sigma_e(L)$ it is important to verify the results (eq. 5) by first calculating $\sigma_e(L)$. Here too we use a circular disc for the sensitive volume.

In a similar way to the effective flux approach which uses an averaged flux [7], we calculate the averaged cross section $\sigma_e(L)$ using similar approximations. Like there, an SEU will occur if $\varepsilon > \varepsilon_c$ or, for a given LET, if $\theta > \theta_c$. For $L \geq L_c$ (meaning: events at all angles) we take $\theta_c = 0$. For smaller LETs, $\theta_c = arccos(L/L_c)$ and for $L \leq \varepsilon_c/\rho a$, $\theta_c = \pi/2$ (i.e. no event even at grazing angles). Note that θ_c depends on the ion (its L) whereas ε_c is a property of the device. The solid-angle-averaged cross section for a given LET value is given by [9]

$$\sigma_e(L) = \int_{\theta_c}^{\pi/2} \sigma_\theta \ \sin\theta \ d\theta = \frac{S}{2}\cos^2\theta_c = \tag{A1}$$

$$= \frac{S}{2}\left(\frac{L}{L_c}\right)^2 \qquad \text{for} \qquad \varepsilon_c/\rho a < L < L_c$$

$$= 0 \qquad \text{for} \qquad L \leq \varepsilon_c/\rho a$$

$$= S/2 \qquad \text{for} \qquad L \geq L_c \ .$$

Eq. (3) with $f(L) = pL^{-n}$ then yields

$$R_H = C^* \frac{S}{L_c^{\,n-1}} - \frac{p}{2(n-1)} \frac{S}{L_{\max}^{\,n-1}} \qquad \text{for} \quad L_c < L_{\max} \tag{A2}$$

$$R_H = \frac{p}{2(n-3)} \frac{S}{L_c^{\,2}} \left[\left(\frac{a}{cL_c}\right)^{n-3} - \left(\frac{1}{L_{\max}}\right)^{n-3} \right] \text{ for } L_c \geq L_{\max} \tag{A3}$$

with $C^* = p[(a/c)^{n-3} - 1]/[(n-1)(n-3)]$, the same as in eq. (5) which is thus identical to eq. (A2) except for a small factor, $1-(a/c)^2$, in the second term which comes from the influence of the slightly different integration limits in the two ways of integration.

Eq. (A3) shows that, due to ions impinging close to 90°, the actual cutoff is at $(a/c)L_{\max}$ rather than at $L_{\max}$. This shows that devices with large L_c and with a»c still obey the FOM formulation but with a smaller value of C, as found in [1]. These devices should show large angular dependence of the heavy ion cross section.